Herding and Delegated Portfolio Management: The Impact of Relative Performance Evaluation on Asset Allocation

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Abstract

This paper investigates the effect of fund managers' performance evaluation on their asset allocation decisions. We derive optimal contracts for delegated portfolio management and show that they always contain relative performance elements. We then show that this biases fund managers to deviate from return-maximising portfolio allocations and follow those of their benchmark (herding). In many cases the trustees of the fund who employ the fund manager prefer such a policy. We also show that fund managers in some situations ignore their own superior information and "go with the flow" in order to reduce deviations from their benchmark. We conclude that incentive provisions for portfolio managers are an important factor in their asset allocation decisions.

1. Introduction

This paper investigates the impact of fund managers' performance evaluation on their asset allocation decisions. An increasing proportion of assets are managed by large institutions, in particular pension funds and mutual funds.[1] Hence, a large proportion of shares and other assets traded on public exchanges are managed by fund managers who are employees and therefore subject to continuous appraisal by the market. This is reflected in the compensation they receive from the owners and trustees of the funds they manage.

In many cases this has introduced strong elements of either explicit or implicit relative performance evaluation into their compensation. Explicit relative performance typically takes the form of benchmarking of returns on the portfolios under management with the return earned by an index or the median fund in the industry. Implicit relative performance evaluation, on the other hand, comes into play when decisions of contract renewal and reallocation of assets under management take into account the performance of other funds over the same period.[2]

This has given rise to a general impression that because the fund managers are evaluated against their peers, they tend to ignore their own information and "go with the flow". In this paper we show that the relative performance element in the fund manager's contract indeed induces them to neglect a part of their own information and adjust their portfolio allocation to that of other funds. In the UK there exists some evidence of remarkable similarity of

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[1] In 1955 institutions owned 23% of US equities while individuals owned 77%; by 1990 the institutional share has gone up to 53% and the share of the individual investors has fallen to 47%. See Lakonishok et al (1991). In UK the equity ownership by institutional investors constitutes about 80% according to Quality of Markets Quarterly (1991) of the London Stock Exchange.

[2] Lakonishok et. al. (1992) observe that advisors who outperform their peers get more new accounts and assets under management. Huddart (1995) models this competition for assets under management.

asset allocations of actively managed portfolios.[3] For Germany, Oehler (1995) has found evidence of herding for mutual funds.

One may argue that ignoring one's own information and mimicking the trades of others may not be too bad if others have superior information. The important question seems to be whether this happens even when one's information is superior. To capture this notion, we consider a case when the fund manager is better informed than the agent he is benchmarked against. If one draws upon the well-accepted terminology in the herding literature, one can think of this as the case where the fund manager is "smart" while the agent who he is benchmarked against is "dumb". We show that with a relative performance contract, it is optimal for the smart manager to neglect his superior information and herd with the dumb, i.e., mimic the trades of the agent against whom he is benchmarked, *even if that agent is less informed*.

We also show that benchmarking can induce the smart manager to herd with the dumb and trade in a direction that is completely opposite of that dictated by his superior information, i.e., buy when he would have sold had he been trading on his own account (and vice-versa). In addition, benchmarking can also encourage the smart manager to trade at times he would not have traded had he been managing his own money. This phenomenon where the smart manager mimics the trades of the dumb is exactly opposite that observed in Scharfstein and Stein (1990) and arises in our model due to the relative performance element in the fund manager's contract.

Unlike previous authors who studied this subject, this paper analyses the economics of asset allocation decisions in the context of a model where the optimal contracts of fund managers are derived explicitly in a principal-agent framework. We analyse two types of con-

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[3] See e. g. the publication by the WM company (1995) on some of the interquartile ranges of UK pension

tracts: (1) moral hazard contracts, where compensation contracts induce the fund manager to make non-observable investments in order to acquire superior information, and (2) screening contracts, where the compensation offered to the fund manager ensures that applicants are of sufficient quality. Under these circumstances, the fund manager is subjected to risk. Since other traders or fund managers in the market are subject to very similar exogenous influences, and perform very similar tasks, the principal can reduce the agent's exposure to risk by making his the fund manager's compensation contingent on his performance relative to his peers. As a consequence, asset allocation decisions of fund managers depend not only on the expected returns and risks of their own portfolio, but also on their (perceived) correlation with those portfolios they are compared with.

Next, we take the relative performance contract as given and examine the fund manager's trading behavior in two different cases. In the first case, we endow the fund manager with superior information and show that at times he ignores his superior information and mimics the trades of the less informed (this is the case discussed above). In the second case, we make the choice of information acquisition endogenous and demonstrate several results. In particular, we show that the fund managers may decide to acquire the same information and the same assets as their peers, even if these decisions do not maximise risk adjusted expected returns (herding). Trustees who write these contracts rationally anticipate this behaviour. Under some circumstances they prefer the wage reduction possible with a relative performance contract to the higher returns available if fund managers do not herd.

The remainder of the paper is structured as follows. Section 2 discusses the related literature. Section 3 introduces the general structure of the model. Section 4 studies two differ-

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funds and *"Fund managers seen to follow herd"*, Financial Times, March 13, 1995, p. 8.

ent reasons for giving fund managers relative performance contracts and derives the optimal contracts. Section 5 takes the optimal contract as given and examines the asset allocation decisions when information endowment is exogenous. Section 6 endogenizes the information acquisition decision and examines the circumstances under which the fund managers decide to acquire the same information as their peers. Section 7 concludes offering suggestions for further research. All proofs are gathered in the appendix.

2. Discussion of the literature

Our work has some similarity with the work of Froot, Scharfstein and Stein (1992). In Froot et. al. investors with a short horizon seek information held by other traders. They ignore information about the fundamental value of the asset and herd on a subset of information. In their work, this happens due to the fact that fundamental information (which has a long term horizon) may not be incorporated into prices before the end of their investment horizon. Whereas they assume exogenously given investment horizons, we derive the optimal contract which gives rise to this suboptimal behavior endogenously. This suboptimal behavior in our model is in the spirit of Dow and Gorton (1994) who improve on the literature on noise trading[4] by explaining apparent noise trading from rational behavior.

Our work differs from the previous theoretical work and argues that herding arises due to the relative performance nature of the compensation contract offered to the fund managers. It shows that the apparently suboptimal herding behavior by fund managers arises from a fully rational response to their compensation contracts.[5] Our work is consistent with the observation made by Lakonishok, Shleifer and Vishny (1992) in their empirical investigation of US

[4] See, e.g. Black (1986), Trueman (1988), de Long et. al. (1989). Also, see Paul (1995).

Pension fund data that "Managers are evaluated against each other. To avoid falling behind a peer group by following a unique investment strategy, they have an incentive to hold the same stocks as other money managers."

This type of relative performance evaluation is also used by Zwiebel (1995) and Huddart (1995) in models with reputation. In these models, in the first period investors learn about the ability of the managers (by comparing their performance with that of others). Having learnt about the ability of the managers in the first period, the investors update their beliefs and take appropriate action in the second period which introduces relative performance evaluation implicitly. In our model, we specify a contract with relative performance evaluation explicitly to capture similar economics in a single period model and obtain stronger results about the behavioral implications.

Other papers in the theoretical literature have analysed herding in the context of managerial reputation, payoff externalities and informational cascades.[6] Herding and other anomalies of investor decisions are also discussed from a behavioral sciences point of view[7]. The pioneering paper on delegated portfolio management is Bhattacharya and Pfleiderer (1985) which examines the case of direct sale of information. Unlike them, however, we focus on indirect sales of information (cf. also Admati and Pfleiderer (1986), (1990)). In our case, the principal designs an optimal contract. Given the contract the fund manager engages in trades

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5 See Brennan (1993) for asset pricing implications when a large fraction of investors are awarded a relative performance contract.

6 See, e.g., Scharfstein and Stein (1990) and Rajan (1994) for herding due to managerial reputation. Examples of herding arising out of payoff externalities are bank runs by Diamond and Dybvig (1983), information acquisition by Brennan (1990) and optimal timing of trades in the presence of informed trading by Admati and Pfliederer (1988). Herding also arises in the context of informational cascades as shown by Banerjee (1992), Welch (1992) and Bickchandani et. al. (1992).

7 See, e.g., de Bondt and Thaler (1985), (1987), Shleifer and Summers (1989), De Long et. al. (1989), Goetzmann and Peles (1994), Shiller and Pound (1989). Kirman (1993) has even borrowed concepts from ecological science to approach this problem. This list is meant to be indicative and not exhaustive. See Devenow and Welch (1995) for a detailed survey of literature on herding.

that maximize his utility. We show that the fund manager's trades differ a great deal from what the principal would have traded had the information been sold directly. Heinkel and Stoughton (1994) and Huddart (1995) have looked at the dynamic implications of delegated portfolio management contracts which are beyond the scope of our paper.

3. The model

The model is a two stage model in which a principal hires an agent (fund manager) to manage her portfolio. In subsequent sections we will describe several variations of the basic model, and this section describes only the general framework common to all of them.

The principal hires an agent who has the ability to acquire information about assets which the principal does not have. After negotiating the initial contract the agent becomes informed about the asset and makes portfolio decisions on behalf of the principal. Generally, we denote the asset payoff by $\tilde{\mathrm{v}}$ where the unconditional distribution of returns is Normal with mean zero and variance σ_{v}^2. The agent learns a signal $\tilde{\mathrm{s}}$ and becomes completely informed ($\tilde{\mathrm{v}} = \tilde{\mathrm{s}}$) or he becomes imperfectly informed. His posterior has mean $\mathrm{E}[\tilde{\mathrm{v}}|\tilde{\mathrm{s}}]$ and $\mathrm{Var}(\tilde{\mathrm{v}}|\tilde{\mathrm{s}}) \equiv \Sigma$ where $\Sigma = 0$ if he is perfectly informed.

The agent makes a portfolio decision by purchasing a quantity x_1 of the asset subsequent to learning $\tilde{s}_1$. The order is placed with a market maker. The order is executed at some price p set by a market maker. Finally, total profits on the portfolio are realised and given by $\tilde{\pi}_1 = x_1(\tilde{v} - p)$. We assume that there exists another trader in the market who trades on his own account. This trader is indexed *2.* Trader 2 purchases a quantity x_2 and realises profits $\tilde{\pi}_2 = x_2(\tilde{v} - p)$.[8]

Since the profits of the agent and trader 2 are correlated, the principal can benefit from this correlation by proposing a relative performance contract to the agent. We assume linearity, so that the generic form of the contract proposed by the principal to the agent is:

$$\tilde{w}_1 = \phi + \alpha\tilde{\pi}_1 - \beta\tilde{\pi}_2 \tag{3.1}$$

where $\tilde{w}_1$ denotes the wage received by trader 1. We assume that both the agent and trader 2 have standard CARA preferences with risk parameter ρ. Then expected utility of the agent conditional on the signal $\tilde{s}_1$ is given as:

$$\begin{aligned} &E\left(U(\tilde{w}_1)\middle|\tilde{s}_1\right) = E\left(\tilde{w}_1\middle|\tilde{s}_1\right) - \frac{\rho}{2}Var\left(\tilde{w}_1\middle|\tilde{s}_1\right) \\ &= \phi + \alpha E\left(\tilde{\pi}_1\middle|\tilde{s}_1\right) - \beta E\left(\tilde{\pi}_2\middle|\tilde{s}_1\right) \\ &- \frac{\rho}{2}\left(\alpha^2 Var\left(\tilde{\pi}_1\middle|\tilde{s}_1\right) - 2\alpha\beta Cov\left(\tilde{\pi}_1,\tilde{\pi}_2\middle|\tilde{s}_1\right) + \beta^2 Var\left(\tilde{\pi}_2\middle|\tilde{s}_1\right)\right) \end{aligned} \tag{3.2}$$

and similarly for unconditional expected utility. The contract is proposed by the principal at the time when the agent is uninformed, so only the unconditional moments of $\tilde{w}_1$ are relevant. We denote these by $V_1 = Var(\tilde{\pi}_1)$, $V_2 = Var(\tilde{\pi}_2)$ and $C_{12} = Cov(\tilde{\pi}_1,\tilde{\pi}_2)$. The principal's objective is to maximise:

$$E\left(\tilde{\pi}_1 - \tilde{w}_1\right) = E\{\tilde{\pi}_1 - (\phi + \alpha\tilde{\pi}_1 - \beta\tilde{\pi}_2)\} = (1-\alpha)E\left(\tilde{\pi}_1\right) + \beta E\left(\tilde{\pi}_1\right) - \phi \tag{3.3}$$

subject to constraints which ensure that the agent accepts the contract proposed and that the agent becomes informed subsequent to accepting the contract.

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[8] One can think of the first agent as a pension fund manager appointed by the trustees while the second

4. Optimal contracts

This section derives the general contract, which is independent of the market mechanism of the second stage. The details of the contract depend on the characteristics of the principal-agent problem. We investigate two situations: one where the agent can only become informed if he incurs a non-verifiable cost *c* (moral hazard, section 4.1), and one where the principal has to screen agents who have the ability to become informed from those who do not (adverse selection, section 4.2).

4.1 Optimal Contracts in the presence of Moral Hazard

This section models hiring of the agent (fund manager) as a moral hazard problem. Subsequent to accepting the contract the agent incurs a non-verifiable cost *c* for obtaining information. Hence, if he becomes informed and trades, his utility is denoted by U_I and given as:

$$U_I = \phi - c + \alpha E\pi_1 - \beta E\pi_2 - \frac{\rho}{2}\left(\alpha^2 V_1 - 2\alpha\beta C_{12} + \beta^2 V_2\right) \tag{4.1}$$

If the agent does not become informed, he does not trade, whereas the second agent (trader 2) continues to trade. Assume the distribution of trader 2's profits is unaffected by the actions of trader 1. Then the utility of agent 1 denoted by U_N if he does not incur the costs *c* to become informed is given by:

$$U_N = \phi - \beta E\left(\tilde{\pi}_2\right) - \frac{\rho}{2}\beta^2 V_2 \tag{4.2}$$

Then the principal's problem for choosing the optimal contract is to maximise the objective (3.3) subject to the participation constraint (PC) that the agent accepts the contract

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trader as someone who is managing the funds of his own company, e.g., an Insurance Company.

proposed, i.e., $U_I \geq 0$, and the incentive compatibility constraint (IC) that the agent becomes informed subsequent to accepting the contract, i.e. $U_I \geq U_N$. Hence, the principal's programme is:

$$\begin{aligned} &\mathrm{Max}_{\{\phi,\alpha,\beta\}} \ (1\text{-}\alpha)\mathrm{E}\pi_1 + \beta \mathrm{E}\pi_2 - \phi \\ \text{s. t. (PC)} \ & U_I \geq 0 \\ \text{(IC)} \ & U_I \geq U_N \end{aligned} \tag{4.3}$$

Note first that the participation constraint is always binding. Otherwise $U_I>0$ and the principal could increase the maximand by reducing ϕ by a small enough amount so that the participation constraint is still satisfied. Since the incentive compatibility constraint cannot be violated by this operation (as both U_I and U_N change by the same amount), the smaller value of ϕ improves on the original solution, thus proving that the participation constraint must be binding.

Substituting the participation constraint into the objective function (i.e., substituting $\phi + \alpha E\pi_1 - \beta E\pi_2 = c + \frac{\rho}{2}\left(\alpha^2 V_1 - 2\alpha\beta C_{12} + \beta^2 V_2\right)$ in equation (4.3)) gives the Lagrangian:

$$\begin{aligned} \mathcal{L} \equiv \ & E(\pi_1) - c - \frac{\rho}{2}\left(\alpha^2 V_1 - 2\alpha\beta C_{12} + \beta^2 V_2\right) \\ & + \psi\left(\alpha E(\tilde{\pi}_1) - c - \frac{\rho}{2}\left(\alpha^2 V_1 - 2\alpha\beta C_{12}\right)\right) \end{aligned} \tag{4.4}$$

where the ψ is the Lagrange multiplier and the term in large brackets equals U_I-U_N. Proposition 1 gives the optimal contract for this case:

Proposition 1 (Moral hazard): *The optimal contract is given by the saddle point of the Lagrangian (4.4). The parameters are:*

$$\beta = \alpha(1+\psi)\frac{C_{12}}{V_2}$$

$$\alpha = \frac{E(\pi_1) - \sqrt{\left(E(\tilde{\pi}_1)\right)^2 - 2\rho c\left(1 - 2(1+\psi)R_{12}^2\right)}}{\rho V_1\left(1 - 2(1+\psi)R_{12}^2\right)} \tag{4.5}$$

where R_{12}^2 *is the R-squared of the regression of the profits of trader 1 on the profits of trader 2.*

Note that the shape of the relative performance contract is determined by the correlation between the profits of the two traders and the cost *c*. The relative performance component of the contract can be measured by β which is proportional to C_{12} / V_2, the slope coefficient of the regression of the profits of trader 1 on the profits of trader 2. α is also related to the R^2 of this regression (being the root of a quadratic equation). If this equation had no real roots (i.e. if the expression under the square root sign in (4.5) is negative), then the incentive compatibility constraint would either never be binding or there would be no α for which the constraint were satisfied. Clearly, the constraint is binding, otherwise α=0 could be a solution, and this is not the case.

4.2 Optimal Contracts as Screening Devices

This section derives the optimal contract when the principal-agent problem involves *screening* good agents from bad agents. Specifically, there are two types of agents. A large number of agents who do not have the ability to process information and become informed (bad agents). Their opportunity costs of being employed are given by c_N. Then there is a small number of agents who have the ability to become informed (good agents). Their opportunity costs of being employed are c_I. We denote the utility of good agents if they accept the contract by U_I where:

$$U_I = \phi - c_I + \alpha E\pi_1 - \beta E\pi_2 - \frac{\rho}{2}\left(\alpha^2 V_1 - 2\alpha\beta C_{12} + \beta^2 V_2\right) \tag{4.6}$$

while we denote the utility of bad agents if they accept the contract by:

$$U_N = \phi - c_N - \beta E\tilde{\pi}_2 - \frac{\rho}{2}\beta^2 V_2 \tag{4.7}$$

Then the principal's problem for choosing the optimal contract (α,β) is to maximise net profits subject to the participation constraint (PC) for the good agents and the screening constraint (SC) whereby the bad agents do not accept the contract. Thus, the principal's programme in this case can be written as:

$$\begin{aligned} &Max_{\{\phi,\alpha,\beta\}}\ (1\text{-}\alpha)E\pi_1 + \beta E\pi_2 - \phi \\ &s.\,t.\ (\text{PC})\ \ U_I \geq 0 \\ &\qquad (\text{SC})\ \ U_N \leq U_I \end{aligned} \tag{4.8}$$

The same argument as before leads to the conclusion that the participation constraint is binding, so that SC becomes $U_N \leq 0$ and the resulting Lagrangian is given by:

$$\mathcal{L} \equiv E\pi_1 - c_I - \frac{\rho}{2}\left(\alpha^2 V_1 - 2\alpha\beta C_{12} + \beta^2 V_2\right) + \psi\left(\phi - c_N - \beta E\pi_2 - \frac{\rho}{2}\beta^2 V_2\right) \tag{4.9}$$

Proposition 2 below provides the solution to the principal's programme.

Proposition 2 (Screening): *The optimal contract is given by the saddlepoint of the Lagrangian (4.9). If* $c_I > c_N$*, then:*

$$\begin{aligned} \alpha &= \beta \frac{C_{12}}{V_1} \\ \beta &= \frac{-E(\tilde{\pi}_2) + \sqrt{E(\tilde{\pi}_2)^2 + 2\rho(\phi - c_N)V_2}}{2\rho V_2} \end{aligned} \tag{4.10}$$

and $\phi<c_N$.

If $c_I \leq c_N$, *then* $\phi = c_I$ *and* $\alpha = \beta = 0$.

The utility of the principal is given as:

$$E(\tilde{\pi}_1) - c_I - \frac{\rho}{2}\beta^2 V_2\left(1 - R_{12}^2\right) \tag{4.11}$$

for both cases.

The screening contract is slightly different from the moral hazard contract, even though they share some similar features. If contracts are used for screening, then the primary goal of the principal is to penalise the agent if he underperforms relative to another agent (benchmark), so that it is not worth his while to accept a contract without having the ability to become informed. This is ensured by the screening constraint (SC). It is easy to see that no benchmarking and risk taking is necessary if the opportunity cost of the potentially informed applicants do not exceed those who cannot become informed. In this case the principal chooses simply $\phi=c_I$ and any bad agent does not apply for the contract since it does not cover his outside employment option. This is a rather uninteresting and also implausible case.

Otherwise, since the principal can not distinguish between good and bad agents, she has to impose a risk on the good as well as the bad agent. The good agent requires compensation for this risk taking, hence this risk needs to be minimised. This is achieved through the value of α given above, which is now proportional to the slope coefficient of regressing the profits of the agent on those of trader 2. Note also that the principal's utility increases in the R^2 of this regression, since a higher correlation of π_1 and π_2 allows for more precise benchmarking. If profits are perfectly correlated, benchmarking is perfect which gives a higher insurance possibility. If profits are perfectly correlated, insurance possibilities are perfect and $\alpha=\beta$, i.e., $\tilde{w} = \phi + \alpha(\tilde{\pi}_1 - \tilde{\pi}_2)$.

This completes the first stage of the model wherein we derived the principal-agent contract in the presence of moral hazard and screening problem. We showed that the optimal contract always involves a relative performance component. Having derived the optimal contract, we now proceed to the second stage of the model and examine the effect of such a contract on the behavior of agents (fund managers). We examine this in two types of market structures. Firstly, a quote driven dealership market with a single asset, and secondly an auction order driven market with two assets.

5. Asset Allocation in a Bid-Ask Market with a Single Risky Asset

We consider a pure exchange economy in which there exists a single risky asset and a riskless asset (the numeraire, with zero normalised return), each pays off in units of a single consumption good.[9] There exists a competitive dealership market in which the risky asset can be traded at the posted prices which all market participants take as given.

The economy consists of two (imperfectly) informed risk averse agents and a large number of liquidity traders whose trades are assumed to be independent of prices for simplicity.[10] The first informed agent is the fund manager while the second informed agent (denoted as trader 2) trades on his own account. As discussed earlier, the principal finds it optimal to offer the fund manager a relative performance contract which links his compensation to the

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[9] This economy is similar to that in George, Kaul and Nimalendran (1994) except that we have competitive market makers and price inelastic liquidity traders.

[10] The discussion below shows that this condition could be relaxed, but the demands of liquidity traders cannot be too elastic.

profits of trader 2. As in the previous section, we continue to assume that both informed agents have CARA preferences over consumption of the single good with risk parameter ρ.

To examine whether an optimal relative performance contract induces a fund manager to ignore his superior information and herd with his peers who may be less informed, we consider the case where the fund manager's information set nests that of trader 2. Thus, this is the case where the fund manager is smart, while trader 2, against whom the fund manager is benchmarked, is dumb.

We denote the risky asset payoff by $\tilde{v}$. Let trader 2 observe a signal $\tilde{s}_2$ while the fund manager observes not only $\tilde{s}_2$ but also an additional signal $\tilde{s}_1$ that is independent of $\tilde{s}_2$.[11] After observing $\tilde{s}_2$, trader 2 maximises his expected utility by choosing to hold x_2 shares of the risky asset. Since trader 2 is trading on his own account his risky asset demand is given by

$$x_2 = \frac{E(\tilde{v}|s_2) - p}{\rho\, Var(\tilde{v}|s_2)} = \frac{\hat{v}_2 - p}{\rho \hat{\Sigma}_2} \tag{5.1}$$

where $\hat{v}_2 \equiv E(\tilde{v}|s_2) \equiv \gamma_2 s_2$ and $\hat{\Sigma}_2 \equiv Var(\tilde{v}|s_2)$ and p is the price (viewed by the trader as exogenously given) at which x_2 units of the risky asset can be purchased (or sold short if $x_2<0$). The profits of trader 2 equal $\tilde{\pi}_2 = x_2(\tilde{v} - p)$.

As derived in the previous section, we assume that the fund manager's contract takes the form $\phi + \alpha\tilde{\pi}_1 - \beta\tilde{\pi}_2$, where ϕ, α and β are constants optimally chosen by the principal. The profits of the fund manager are given by $\tilde{\pi}_1 = x_1(\tilde{v} - p)$ where x_1 is his risky asset demand. Since in this economy all market participants take the posted bid-ask prices as given

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[11] The assumption of independence is without loss of generality. One can always orthogonalize correlated signals to obtain the above structure.

and submit demands for the risky asset, we get $\tilde{\pi}_1 = x_1(\tilde{v} - p)$, $\tilde{\pi}_2 = x_2(\tilde{v} - p)$. The variances of profits are given by:

$$
\begin{aligned}
&Var(\tilde{p}_1) = x_1^2 Var(\tilde{v}) \\
&Var(\tilde{p}_2) = x_2^2 Var(\tilde{v}) \\
&Cov(\tilde{p}_1, \tilde{p}_2) = x_1 x_2 Var(\tilde{v})
\end{aligned}
\tag{5.2}
$$

The fund manager observes the realizations s_1 and s_2, takes x_2 as given and maximises the expected utility of his compensation by solving the following problem:

$$
\begin{aligned}
Max_{\{x_1\}} \phi + \alpha x_1 E\{(\tilde{v} - p)|s_1, s_2\} - \beta x_2 E\{(\tilde{v} - p)|s_1, s_2\} \\
- \frac{\rho}{2}\left\{\alpha^2 x_1{}^2 - 2\alpha\beta x_1 x_2 + \beta^2 x_2{}^2\right\} Var(\tilde{v}|s_1, s_2)
\end{aligned}
\tag{5.3}
$$

Lemma 1: *The fund manager's optimal stock demand is obtained from the first order condition as:*

$$
x_1 = \frac{\hat{v}_1 - p}{\alpha\rho\hat{\Sigma}_1} + \frac{\beta}{\alpha} x_2 = \frac{\hat{v}_1 - p}{\alpha\rho\hat{\Sigma}_1} + \frac{\beta}{\alpha}\frac{\hat{v}_2 - p}{\rho\hat{\Sigma}_2}
\tag{5.4}
$$

where $\hat{v}_1 \equiv E(v|\tilde{s}_2, \tilde{s}_1) = \gamma_2 s_2 + \gamma_1 s_1$ *and* $\hat{\Sigma}_1 \equiv Var(v|\tilde{s}_1, \tilde{s}_2)$.

Notice that the first term in equation (5.4) is proportional to the demand had the fund manager been trading on his own account while the second term captures the effect of benchmarking the fund manager with trader 2. Expressing the conditional expectations of the risky asset payoff in terms of the signals, we can write the fund manager's optimal risky asset demand as:

$$
\tilde{x}_1 = \frac{(\gamma_1\tilde{s}_1 + \gamma_2\tilde{s}_2) - p}{\alpha\rho\hat{\Sigma}_1} + \frac{\beta}{\alpha}\frac{(\gamma_2\tilde{s}_2 - p)}{\rho\hat{\Sigma}_2},
$$

$$\tilde{x}_1 = \left\{\frac{\gamma_1}{\alpha\rho\hat{\Sigma}_1}\right\}\tilde{s}_1 + \left\{\frac{\gamma_2}{\alpha\rho\hat{\Sigma}_1} + \frac{\beta\gamma_2}{\alpha\rho\hat{\Sigma}_2}\right\}\tilde{s}_2 - \left\{\frac{1}{\alpha\rho\hat{\Sigma}_1} + \frac{\beta}{\alpha\rho\hat{\Sigma}_2}\right\}p$$
$$\equiv \delta_1\tilde{s}_1 + \delta_2\tilde{s}_2 - \delta_3 p \qquad (5.5)$$

Proposition 3: *For any relative performance contract with $\alpha>0$ and $\beta>0$, the weights the fund manager gives to common information is larger, and that given to his idiosyncratic information is smaller than given by the Bayesian updating rule, i.e., $\delta_1/\delta_2 < \gamma_1/\gamma_2$.*

Hence, for any relative performance contract there is some herding in the sense that the fund manager always puts a disproportionately larger weight on information he has in common with the trader against whom he is benchmarked, and a disproportionately smaller weight on information private to him. The critical parameter here is β, since if β=0, the demand sensitivities γ_1, γ_2 would be exactly proportional to δ_1, δ_2 (i. e. $\gamma_1 / \gamma_2 = \delta_1 / \delta_2$), the parameters of the updating rule.

The optimal stock demands derived above implicitly assumed that the risky asset can be bought or sold at the same price p. We now recompute their optimal stock demands with a given ask (bid) price at which the risky asset can be purchased (sold). Let the posted ask price be $p_{ask} = \bar{p} + \bar{s}$ and the bid price be $p_{bid} = \bar{p} - \bar{s}$.[12] In general, given the normal distribution of signals, $\hat{v}_2 = \gamma_2 s_2$ can be greater than the ask price, less than the bid price or can lie within the bid and the ask prices. In the presence of bid-ask spread, the optimum stock demand of trader 2 is given by:

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[12] We show later that there exists a unique equilibrium bid-ask spread in this economy.

$$x_2 = \begin{cases} \dfrac{\hat{v}_2 - (\bar{p} + \bar{s})}{\rho\hat{\Sigma}_2} & if\, \hat{v}_2 > \bar{p} + \bar{s} \\ \dfrac{\hat{v}_2 - (\bar{p} - \bar{s})}{\rho\hat{\Sigma}_2} & if\, \hat{v}_2 < \bar{p} - \bar{s} \\ zero & otherwise \end{cases} \tag{5.6}$$

Since the fund manager observes an additional signal $\tilde{s}_1$, in general, $\hat{v}_1 = \gamma_1 s_1 + \gamma_2 s_2$ differs from $\hat{v}_2$. Furthermore, since signals $\tilde{s}_1$ and $\tilde{s}_2$ are normally distributed and independent of each other, for any given realisation of $\hat{v}_2$, $\hat{v}_1$ can be greater than the ask price, less than the bid price or can lie within the bid and ask prices. Thus, in total we get nine possible scenarios. Table I below describes these nine possibilities along with the optimal stock demand of the fund manager (equation (5.4) duly adapted to take into account the bid-ask spread and the piecewise linearity of x_2) in each of the cases.

TABLE I

	$\hat{v}_1 > p_{ask}$	$\hat{v}_1 < p_{bid}$	$p_{bid} < \hat{v}_1 < p_{ask}$
$\hat{v}_2 > p_{ask}$	Fund Manager buys more aggressively.	Fund Manager buys, sells or does not trade depending on strength of signal.	Fund Manager may buy or not trade, depending on strength of signal.
$\hat{v}_2 < p_{bid}$	Fund Manager buys, sells or does not trade depending on strength of signal.	Fund Manager sells more aggressively.	Fund Manager may sell or not trade, depending on strength of signal.
$p_{bid} < \hat{v}_2 < p_{ask}$	Fund Manager buys as if β=0.	Fund Manager sells as if β=0.	Fund Manager does not trade (which is the optimal action if β=0).

Consider, for instance, the first row of Table I. This corresponds to the case where the conditional expectation of trader 2 is above the ask price (i.e., $\hat{v}_2 > p_{ask}$ or $\tilde{s}_2 > \frac{\overline{p}+\overline{s}}{\gamma_2}$) and, according to equation (5.6), trader 2 would be buying the risky asset. However, for any realization of $\tilde{s}_2$ satisfying this condition, there would be realizations of $\tilde{s}_1$ which would make $\hat{v}_1$ greater than the ask price, less than the bid price or in between the bid and the ask price. If the fund manager would be trading on his own account, then he would have bought the stock in the first case, sold the stock in the second and not traded at all in the third case, i.e., the fund manager's stock demand would have looked like equation (5.6). However, due to the relative performance nature of his contract, the fund manager's stock demand turns out to be different. This can best be seen in figure 1.

[Insert Figure 1 about here]

In figure 1, all combinations of $\hat{v}_1$ and $\hat{v}_2$ are shown in $\tilde{s}_1 - \tilde{s}_2$ space. Since the second trader does not observe $\tilde{s}_1$, the regions corresponding to his decision rules are separated by horizontal lines parallel to the $\tilde{s}_1$-axis. For instance, the region where $\hat{v}_2 > p_{ask}$ is above the horizontal broken line intersecting the $\tilde{s}_2$-axis at p_{ask} / γ_2. Similarly, $\hat{v}_2 < p_{bid}$ is below the line parallel to this intersecting at p_{bid} / γ_2. In the region between the horizontal broken lines, $p_{bid} < \hat{v}_2 < p_{ask}$. Hence, the three regions between the two horizontal broken lines correspond to the rows of table I.

Case (I): $\hat{v}_1 > p_{ask}$ and $\hat{v}_2 > p_{ask}$. This is described by $\tilde{s}_2 > \frac{\overline{p}+\overline{s}}{\gamma_2}$ and $\tilde{s}_1 > \frac{\overline{p}+\overline{s}}{\gamma_1} - \frac{\gamma_2}{\gamma_1}\tilde{s}_2$ (case Ia). In this case both terms in (5.4) are positive, and it follows that in this case the fund manager buys more aggressively compared to what he would have traded if his contract did not have a relative performance element (β=0). In figure 1, the region where $\hat{v}_1 > p_{ask}$ lies above and right of the diagonal broken line which has slope $-\gamma_2 / \gamma_1$ and intersects the $\tilde{s}_2$-axis at p_{ask} / γ_2. Conversely, if $\hat{v}_1 < p_{bid}$ and $\hat{v}_2 < p_{bid}$. This is described by $\tilde{s}_2 < \frac{\overline{p} - \overline{s}}{\gamma_2}$ and $\tilde{s}_1 < \frac{\overline{p} - \overline{s}}{\gamma_1} - \frac{\gamma_2}{\gamma_1}\tilde{s}_2$ (case Ib). In this case both terms in (5.4) are negative, and it follows that in this case the fund manager sells more aggressively compared to the case where β=0. In figure 1, the region where $\hat{v}_1 < p_{bid}$ lies below and to the left of the lower broken line with slope $-\gamma_2 / \gamma_1$ which intersects the $\tilde{s}_2$-axis at p_{abid} / γ_2. In the region between the diagonal broken lines, $p_{bid} < \hat{v}_1 < p_{ask}$. Hence, the three regions separated by the diagonal broken lines correspond to the columns of table I.

Case (II): Consider now the case where $\hat{v}_1 < p_{bid}$ and $\hat{v}_2 > p_{ask}$ (case IIa). This region in figure 1 is defined by $\tilde{s}_2 > \frac{\overline{p} + \overline{s}}{\gamma_2}$ and $\tilde{s}_1 < \frac{\overline{p} - \overline{s}}{\gamma_1} - \frac{\gamma_2}{\gamma_1}\tilde{s}_2$. In this case the two terms in (5.4) are of opposite sign, and the fund manager has three choices: to sell (follow his own, superior information), to buy (herd with the trader he is benchmarked against), or not to trade. The fund manager's expected utility under no trade is given by:

$$E\{U(x_1 = 0)\} = \phi - \beta x_2(\hat{v}_1 - p) - \frac{\rho}{2}\beta^2 x_2^2 \hat{\Sigma}_1, \tag{5.7}$$

while his expected utility when he submits the optimal demand is given by:

$$E\{U(x_1)\} = \phi + (\alpha x_1 - \beta x_2)(\hat{v}_1 - p) - \frac{\rho}{2}\{\alpha^2 x_1^2 - 2\alpha\beta x_1 x_2 + \beta^2 x_2^2\}\hat{\Sigma}_1 \tag{5.8}$$

By trading in this case, the fund manager expects to make a loss on his portfolio but also to reduce his risk. He herds with trader 2 and buys the stock if the expected utility of trading turns out to be higher than that of no trade. The optimal decision is expressed in the following lemma:

Lemma 2: *The fund manager prefers to trade in the stock and buy (sell), rather abstain from trading, whenever:*

$$\frac{\hat{v}_1 - p_{ask}}{\alpha\rho\hat{\Sigma}_1} + \frac{\beta}{\alpha}\frac{\hat{v}_2 - p_{ask}}{\rho\hat{\Sigma}_2} > 0 \quad (< 0) \tag{5.9}$$

This is intuitive and tells us that the fund manager will herd with the less informed trader and buy (sell) the risky asset even when $\hat{v}_1 < p_{bid}$ $(\hat{v}_1 > p_{ask})$ provided his own information is not too strong. Hence, the optimal demand given by equation (5.4). As a consequence, the fund manager will sell if:

$$\tilde{s}_1 < \left(\frac{p_{Bid}}{\gamma_1} - \frac{\gamma_2}{\gamma_1}\tilde{s}_2\right)\left(1 + \beta\frac{\hat{\Sigma}_1}{\hat{\Sigma}_2}\right) \tag{5.10}$$

and buy if:

$$\tilde{s}_1 > \left(\frac{p_{Ask}}{\gamma_1} - \frac{\gamma_2}{\gamma_1}\tilde{s}_2\right)\left(1 + \beta\frac{\hat{\Sigma}_1}{\hat{\Sigma}_2}\right) \tag{5.11}$$

and not trade if $\tilde{s}_1$ is between these two values. Conversely, if $\hat{v}_1 > p_{ask}$ and $\hat{v}_1 < p_{bid}$ the two terms in (5.4) are again of opposite sign, but in the reverse order (Case IIb). This region in figure 1 is defined by $\tilde{s}_2 < \dfrac{\overline{p} - \overline{s}}{\gamma_2}$ and $\tilde{s}_1 > \dfrac{\overline{p} + \overline{s} - \gamma_2\tilde{s}_2}{\gamma_1}$. The trading decisions are again given by equations (5.10) and (5.11) as before. The regions describing the trading decisions of the fund manager are represented by the diagonal solid lines in figure 1 (the upper line corresponds to (5.11), the lower one to (5.10)). The slope of the solid lines equals

$-\frac{\gamma_2}{\gamma_1}\left(1+\beta\frac{\hat{\Sigma}_1}{\hat{\Sigma}_2}\right)$ in $\tilde{s}_2-\tilde{s}_1$-space, hence the slopes are *less* steep than those of the broken diagonal lines in $\tilde{s}_1-\tilde{s}_2$-space.

Hence, in this case the fund manager knows that the less informed trader has information which biases him in the opposite direction as his own, superior information. As a consequence, the fund manager will either (a) herd with the less informed trader and buy (sell) the stock where he would otherwise have sold (bought) *(i.e., take a position completely opposite of that dictated by his information),* or (b) sell an amount which is less than he would have sold if his contract did not have a relative performance element, or (c) not trade at all. In figure 1 these cases correspond to the two conical regions (1) below the lower broken diagonal line and above the upper solid diagonal line (fund manager's information indicates to sell, but he buys), and (2) above the upper broken diagonal line and below the lower solid diagonal line (fund manager's information indicates to buy, but he sells).

Case (III): Now consider the case where $p_{bid} < \hat{v}_1 < p_{ask}$. This region is defined and $\frac{\bar{p}-\bar{s}-\gamma_2\tilde{s}_2}{\gamma_1} < \tilde{s}_1 < \frac{\bar{p}+\bar{s}-\gamma_2\tilde{s}_2}{\gamma_1}$. In this case the fund manager would not trade on the basis of his own information if he had no relative performance contract. However, if $\hat{v}_2 > p_{ask}$ (hence: $\tilde{s}_2 > \frac{\bar{p}+\bar{s}}{\gamma_2}$), the other trader buys, and he will be induced to herd and buy if condition (5.11) is satisfied. Conversely, if $\hat{v}_2 < p_{bid}$ (hence: $\tilde{s}_2 < \frac{\bar{p}-\bar{s}}{\gamma_2}$), the other trader sells, and he will be induced to sell if condition (5.10) is satisfied. In figure 1, the region where $p_{bid} < \hat{v}_1 < p_{ask}$ lies between the diagonal broken lines. The strip above the upper solid line is the region where the fund manager buys, and the region below the lower solid line where he sells, even though his information indicates that these trades are unprofitable in both cases.

This completes the discussion of all nine possibilities listed in table I. As can be seen, the fund manager's optimal stock demand turns out to be very different from what the fund manager would have traded had he been trading on his own account. Having characterized the optimal risky asset demands of trader 2 and the fund manager, we now solve for the equilibrium bid-ask spread $2\bar{s}$ in this economy.

Recall that there exist liquidity traders who submit random trades in the risky asset which are independent of $\tilde{v}$. We assume that the trades of the liquidity traders are insensitive to the bid-ask spread.[13] Let $\tilde{L}_{ask}$ denote the liquidity demand for buying the stock and $\tilde{L}_{bid}$ the liquidity demand for selling the stock. Thus, the total volume of liquidity trading equals $|\tilde{L}_{ask}|+|\tilde{L}_{bid}|$. Define $\tilde{L} = \tilde{L}_{ask} + \tilde{L}_{bid}$ and denote the mean of this distribution by $\bar{L}$.

We further assume that there exists a competitive dealership market with N market makers who know the structure of the economy- the composition of agents in the economy, the nature of their trading rules and the parameters of distributions describing uncertainty. We assume that each market maker obtains an equal share of liquidity trading and has an equal chance of getting the informed trades. We also assume that potential competition for market making drives the bid and ask prices to a level which offers zero expected profits to the market maker in equilibrium.

Recall that $\bar{s}$ denotes one half of bid-ask spread and let $\bar{p}$ be the mid-point of posted bid-ask prices. Then, for each share of the risky asset bought or sold, the market maker earns one half of the spread. If the asset is underpriced relative to its true payoff, the market maker

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13 George, Kaul and Nimalendran (1994) solve for an equilibrium in a similar economy when the spread is set by profit maximizing specialist and the liquidity traders submit random but spread sensitive trades. Unlike our economy, there may not exist an equilibrium in theirs. However, whenever there does exist an equilibrium, our results hold in their economy as well.

makes money if market participants sell the asset and loses money if they buy the asset. Let $\tilde{x}_1$ and $\tilde{x}_2$ be the risky asset demands of the fund manager and trader 2 (worked out above) as a function of the bid and the ask prices. Then, the profit of each market maker is given by:

$$\tilde{\pi} = \frac{1}{N} s(|\tilde{L}_{ask}| + |\tilde{L}_{bid}| + |\tilde{x}_1| + |\tilde{x}_2|) - \frac{1}{N} (\tilde{v} - \tilde{p})(\tilde{x}_1 + \tilde{x}_2 + \tilde{L}) \tag{5.12}$$

When the spread equals zero, each market maker makes a loss. As the spread starts to increase, the losses from trading with the informed agents start to reduce while the profits from the liquidity trades start to increase. When the spread becomes very large, each market maker expects to earn a positive profit. Thus, $E(\tilde{\pi})$ is monotone increasing in the bid-ask spread and there exists a unique equilibrium spread $\bar{s}$ in which provides each market maker earns zero expected profit. Thus, we have just shown:

Lemma 3: *There always exists a unique equilibrium bid-ask spread $2\bar{s}$, so that each market maker earns zero expected profits.*

This completes the description of the equilibrium in the economy under consideration. In this economy, unconditionally the profits of trader 2 and the fund manager are positively correlated due to the common element of the signal. A positive correlation between the profits implies that ex-ante it is optimal for the principal to offer a relative performance contract of the form $\phi + \alpha\tilde{\pi}_1 - \beta\tilde{\pi}_2$ where the contract parameters are chosen as shown in the previous section. Hence, summarizing the discussion we have:

Proposition 4: *There exists a unique equilibrium in which all market makers earns zero expected profits, the principal offers and the fund manager accepts a relative performance contract (β>0), and there is a positive probability that $\hat{v}_1 > p_{ask}$ and the fund* *manager sells, or that* $\hat{v}_1 < p_{bid}$ *and he buys in order to stay closer to the portfolio allocation of trader 2.*

Hence, in this section we have shown that the optimal contract chosen by the principal induces some degree of herding. This can take either the form of more aggressive trading if the fund manager shares the view of the trader he is benchmarked against, or of trading in the same direction as his benchmark. The relative performance contract sometimes induces the fund manager to trade in a direction that is completely opposite of what he would have done had he been trading on his own account, simply in order to "go with the flow". Sometimes, it also induces the fund manager to trade when he would not have traded had he been managing his own money. Even though there is a positive probability of these events happening, it is optimal for the principal to offer a relative performance contract ex ante in order to save on the costs of screening or providing incentives to the agent. However, ex-post it leads to inefficient outcomes with positive probability.[14] This leads to a situation where the trustees of the fund offer the fund manager a contract in order to induce him to become informed. However, they offer this contract on terms which induce the fund manager to partly ignore the information he has thus acquired.

Until now we focused on the case where the information structure was specified exogenously. In the next section, we examine a case where the fund manager choses to acquire information endogenously.

[14] If both informed agents have identical information, then the fund manager does not get subjected to any risk and wishes to trade an arbitrarily large quantity. In such a case, there does not exist an equilibrium in this economy. If trader 2's information set nests that of the fund manager, then the fund manager considers his conditional expectation to be that of the other trader as well and submits optimal trades given his information..

6. Asset Selection in an Auction Market

This section describes an alternative market structure for the second stage of the model and examines the case where there are two assets, indexed *A* and *B*. The agent can potentially become perfectly informed about either one but not both. One can think of the two assets as stocks of firms in two different countries. The agent, due to say time constraint, can only acquire information about one of the two but not both. The returns on the two assets are correlated. Assume that the joint distribution of asset returns is given by:

$$\begin{pmatrix} \tilde{v}_A \\ \tilde{v}_B \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_A^2 & r\sigma_A\sigma_B \\ r\sigma_A\sigma_B & \sigma_B^2 \end{pmatrix}\right), \quad r > 0. \tag{6.1}$$

Trader 2 learns $\tilde{v}_B$ perfectly and trades in asset *B* only. The fund manager can choose between learning either $\tilde{v}_A$ or $\tilde{v}_B$ perfectly, but not both. No trader can trade in both markets at the same time. Then, *after* being hired the fund manager can make a decision whether to trade in asset *A* or in asset *B*.

Both assets are traded in a market where both the informed traders and uninformed liquidity traders submit orders to the market maker. The market makers quote a price schedule in which the price at which the transaction is executed depends only on the size of transaction. The price p_i at which the order x_i for asset i, i=A,B is executed is:

$$p_i = \lambda_i x_i \tag{6.2}$$

where λ_i denotes the depth of the market. Standard analysis shows that $x_i = \tilde{v}_i / 2\lambda_i$ and the profits from this transaction are given as $\tilde{\pi}_i = \tilde{v}_i^2 / 4\lambda_i$.

Assume that the distribution of noise traders is given by:

$$\begin{pmatrix} \tilde{u}_A \\ \tilde{u}_B \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{UA}^2 & 0 \\ 0 & \sigma_{UB}^2 \end{pmatrix} \right) \tag{6.3}$$

Now, conjecture an equilibrium in which the fund manager trades in asset A. Also, assume that market making is a competitive industry so that market makers make expected profits of zero in equilibrium in each asset. Then the expected gains the market makers make from trading with noise traders must equal the expected losses from trading with informed traders. The gains from trading with noise traders are in expectation $\lambda_i \sigma_{Ui}^2$, so that $\lambda_i = \sigma_i / 2\sigma_{Ui}$. The moments of the distribution of the profits of informed traders can then be calculated as:

$$\begin{gathered} E(\tilde{\pi}_i) = \frac{\sigma_i^2}{4\lambda_i}, \qquad \mathrm{Var}(\tilde{\pi}_i) = \frac{\sigma_i^4}{8\lambda_i^2} \equiv V_i = 2\left(E(\pi_i)^2\right) \\ \mathrm{Cov}(\tilde{\pi}_A, \tilde{\pi}_B) = \frac{E\left(\left[\tilde{v}_A^2 - \sigma_A^2\right]\left[\tilde{v}_B^2 - \sigma_B^2\right]\right)}{16\lambda_A \lambda_B} \equiv C_{AB} \end{gathered} \tag{6.4}$$

Then the utility of the fund manager if he becomes informed about assets A and B respectively are then:

$$\begin{gathered} U(A) = \phi - c + \alpha E\pi_A - \beta E\pi_B - \frac{\rho}{2}\left(\alpha^2 V_A - 2\alpha\beta C_{AB} + \beta^2 V_B\right) \\ U(B) = \phi - c + (\alpha - \beta) E\pi_B - \frac{\rho}{2}\left(\alpha^2 - 2\alpha\beta + \beta^2\right) V_B \end{gathered} \tag{6.5}$$

Assume that the principal cannot control in which market the agent trades. Then the fund manager will deviate and choose to trade in the same market as trader 2 whenever $U(B) > U(A)$. In order to examine the simplest case first, assume that the distributions of profits for both assets are identical, but they are still imperfectly correlated ($\sigma_A = \sigma_B$, r < 1). Then $E(\tilde{\pi}_A) = E(\tilde{\pi}_B)$ and V_A=V_B. Then the previous equation gives:

$$U(A) - U(B) = \rho\alpha\beta(C_{AB} - V_B) < 0 \;\Leftrightarrow\; \beta > 0 \tag{6.6}$$

This gives rise to the following result:

Proposition 5 (Herding): *Suppose the marginal distributions of asset returns are identical, but they are imperfectly correlated (* $\sigma_A = \sigma_B$, r < 1 *). Then no relative performance contract is viable and any relative performance contract with* β*>0 implies that the fund manager deviates and trades in asset B.*

The intuition for this result is easy to see. If both assets have the same distribution, then the expected profits from trading in either one of them are the same. However, if profits are the same, then the only difference in learning about the asset from the point of view of the agent is the risk term, especially since the variance of profits are the same for both markets - the covariance terms (cf. equation (6.6)). This is another way of expressing the fact that if the fund manager trades in the same market as the trader 2 then he increases his insurance and reduces his risk. Then, any relative performance contract makes him sensitive to this risk and gives him an incentive to trade in the same asset as the trader he is benchmarked against. Since expected profits are the same, he decides on the basis of this insurance possibility alone. This leaves the principal, the trustees of fund, with the following choice.

Option 1: He can accept that both traders trade in the same market. This is then rationally anticipated by the market maker, and now the market maker increases λ_B to $\lambda_B^{'}$ in order to take account of the presence of two informed traders instead of one. The condition is now:

$$2\mathrm{E}(\tilde{\pi}_{\mathrm{A}}) = \lambda_{\mathrm{B}}\sigma_{\mathrm{UB}}^2 \iff \lambda_{\mathrm{B}} = \frac{\sigma_{\mathrm{B}}}{\sigma_{\mathrm{UB}}\sqrt{2}} \tag{6.7}$$

Hence, λ_B has increased by a factor of $\sqrt{2}$. From (6.4) and (6.6) this implies that profits are reduced by this factor.

Option 2: Alternatively, the principal can forego the insurance from the relative performance contract and set β equal to zero. This would give expected profits to the principal as:

$$\mathrm{E}(\tilde{\pi}_{\mathrm{B}}) - \mathrm{c} - \frac{\rho}{2}\alpha^2 \mathrm{V}_{\mathrm{A}} \tag{6.8}$$

Comparing the two options (equations (6.7) and (6.8)) shows then:

Proposition 6 (Equilibrium herding): *Under the assumptions of proposition 5, the principal prefers the fund manager to herd if and only if:*

$$\rho > \frac{\sqrt{2} - 1}{\alpha^2 \mathrm{E}(\pi_{\mathrm{A}})\sqrt{2}} \tag{6.9}$$

where α is given from:

$$\alpha = \frac{1}{2\rho \mathrm{E}(\tilde{\pi}_{\mathrm{B}})}\left(1 - \sqrt{1 - 4\rho \mathrm{c}}\right) \tag{6.10}$$

The result shows that the principal's problem focuses mainly on the degree of risk aversion, i. e. the risk premium she has to pay to the fund manager. The trade-off is essentially between a larger profit in an alternative market A, where the agent does not face any competition, but the lower correlation of returns implies that the agent receives less insurance (option

1), or the principal foregoes the larger expected profit and the agent receives full insurance by trading in the same market (option 2).

The previous discussion has only analysed the simple case where the distribution of profits in the two markets is identical. This section returns to the more general case where they are different. More specifically, assume that $\sigma_A^2 > \sigma_B^2$. In order to show that herding can occur, it is sufficient to derive an upper bound of the profits the principal can obtain by inducing the agent to trade in asset A, and then to show for which parameters the profits from trading in asset B exceed this upper bound.

The profits from trading in asset A to the principal are:

$$E\pi_A - c - \frac{\rho}{2}\left(\alpha^2 V_A - 2\alpha\beta C_{AB} + \beta^2 V_B\right) \qquad (6.11)$$

An upper bound for this expression can be found by minimising the risk term over all β without taking into account the incentive compatibility constraint. This gives $\beta = \alpha C_{AB}/V_B$. Then a result similar to proposition 4 obtains:

Proposition 7: *The principal prefers the fund manager to herd (trade in market B) and give full insurance if:*

$$\rho > \frac{\sqrt{2} - \frac{E(\tilde{\pi}_B)}{E(\tilde{\pi}_B)}}{\alpha^2 E(\tilde{\pi}_A)\sqrt{2}\left(1 - R_{AB}^2\right)} \qquad (6.12)$$

where α is given as in proposition 6 and $R_{AB}^2 = C_{AB}^2 / V_A V_B$ is the R^2 of the regression of $\tilde{\pi}_A$ on $\tilde{\pi}_B$.

The result shows that the intuition behind proposition 6 generalises to the case where profits in market A are *higher* than profits in market B. Note that the bound for ρ is increasing in the R^2 of the regression: the higher the R^2, the better the principal can insure the agent for the risk of trading in another market, thereby reducing the costs for the principal.

7. Concluding Remarks

The paper has studied a model which links asset allocation decisions taken by professional fund managers to their compensation schemes. Optimal compensation schemes were derived for two cases: (1) fund managers have to be provided with incentives to incur private costs of information acquisition (moral hazard), and (2) trustees have to ensure that only sufficiently competent managers apply for a position of a fund manager (screening). Both scenarios lead to relative performance contracts. In a second step the implications of these contracts for portfolio decisions were derived in the context of different market structures. The first set up analysed the case where one risky asset was traded in a bid-ask market and the agents had imperfect but nested information sets. The main results were:

- Fund mangers adjust their portfolio allocation to that of other funds, even if this implies that they neglect their own superior information.
- Neglecting their superior information may even take the form of buying where they would have sold (and vice versa) if they were trading on their own account.
- Sometimes the fund managers herd and trade when they would not have traded had they been managing their own money.

The second set-up of market structure analysed the choice between two different assets if the fund manager can acquire perfect information about one of these assets, and all fund managers have access to the same information. It was shown that:

- Fund managers decide to acquire the same information and the same asset as their peers, even if the expected returns from these decisions are smaller.
- Trustees who write these contracts rationally anticipate this behaviour. Under some circumstances they prefer the insurance possibilities of a relative performance contract to the higher returns available if fund managers do not herd.

These results could be extended in many ways. For instance, the derivation of optimal contracts was limited to the case where compensation schemes were restricted to be linear. This is an important limitation since optimal delegated portfolio management contracts could be non-linear.[15] However, since a relative performance element would also be part of a more general contract, we believe that an analysis based on more general contractual forms would not lead to different qualitative implications for asset allocations.

We have also restricted attention to the case of one fund manager whose compensation is set through a principal-agent set-up, treating the second fund mangers' compensation as exogenous by assuming that he is trading on his own account. A more general model could look at the implications of relative performance contracts for all funds, effectively benchmarking funds against each other. Richer informational settings, e.g., allowing for non-nested information sets may yield further interesting insights. We leave these questions for future research.

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[15] See, e.g., Bhattacharya and Pfleiderer (1985) and Stoughton (1993) for non-linear contracts.

Appendix

Proof of proposition 1

First order conditions for β are:

$$\rho\ (\alpha\, C_{12} - \beta\, V_2) + \psi\rho\alpha\, C_{12}\ =\ 0 \tag{A 1}$$

$$\Leftrightarrow \beta\ =\ \alpha\ (1+\psi)\ \frac{C_{12}}{V_2} \tag{A 2}$$

Inserting the expression for β into the incentive compatibility constraint gives:

$$\alpha\ E\ \pi_1\ -\ c\ -\ \frac{\rho}{2}\ \alpha^2\ \ V_1\ +\ \ \rho\ \ \alpha^2\ (1\ +\ \ \psi)\ \frac{C_{12}^2}{V_2}\ \geq\ 0 \tag{A 3}$$

If the constraint in binding, α is given by the smallest positive root of the above quadratic equation. This can be rewritten using $R_{12}^2\ \equiv\ C_{12}^2 / V_1 V_2$, the R-squared of the regression of π_1 on π_2:

$$\alpha\ E\,\pi_1\ -\ c\ -\ \frac{\rho}{2}\ V_1\ (1\text{-}2\ (1+\psi)\ \ R_{12}^2)\ \ \alpha^2\ =\ 0 \tag{A 4}$$

which has roots

$$\alpha_{1,2}\ =\ \frac{-E\,\pi_1 \pm \sqrt{(E\,\pi_1)^2 - 2\rho\, c\, V_1 (1\text{-}2(1+\psi)\, R_{12}^2)}}{\rho\, V_1 (1\text{-}2(1+\psi)\, R_{12}^2)} \tag{A 5}$$

Since the objective decreases in α, the smallest root is the solution.■

Proof of proposition 2

First order conditions for α are:

$$\rho\ (\alpha\ V_1 - \beta\ C_{12}) = 0 \Leftrightarrow \alpha = \beta\ \frac{C_{12}}{V_1} \tag{A 6}$$

Inserting this into the objective gives

$$E\ \pi_1 - c_I - \frac{\rho}{2}\ \beta^2\ V_2\ (1 - R_{12}^2) \tag{A 7}$$

Hence, the smallest β that satisfies the screening constraint needs to be chosen. There are two possible cases:

Case 1: $c_I \leq c_N$

Then $\alpha = \beta = 0$ and $\phi = c_I$ satisfies both constraints and maximises the objective.

Case 2: $c_I > c_N$

Then $\beta > 0$ in order to satisfy the screening constraints. Since the objective decreases in β, the constraint must be binding. The solutions to the quadratic are:

$$\beta_{1,2} = \frac{E\ \pi_2 \pm \sqrt{(E\ \pi_2)^2 + 2\ \rho\ (\phi - c_N)\ V_2}}{-2\rho V_2} \tag{A 8}$$

The smaller root is chosen and given in the text.■

Proof of proposition 3:

Note from equation (5.4) that $\delta_1 = \frac{\gamma_1}{\alpha\rho\hat{\Sigma}_1}$ and $\delta_2 = \frac{\gamma_2}{\alpha\rho\hat{\Sigma}_1} + \frac{\beta\gamma_2}{\alpha\rho\hat{\Sigma}_2}$.

Therefore, $\frac{\delta_1}{\delta_2} = \frac{\gamma_1}{\gamma_2}\left(\frac{\frac{1}{\alpha\rho\hat{\Sigma}_1}}{\frac{1}{\alpha\rho\hat{\Sigma}_1} + \frac{\beta}{\alpha\rho\hat{\Sigma}_2}}\right) = \frac{\gamma_1}{\gamma_2}\left(\frac{1}{1 + \frac{\beta\hat{\Sigma}_1}{\hat{\Sigma}_2}}\right)$ and since $\beta, \hat{\Sigma}_1, \hat{\Sigma}_2$ are all positive, it follows that $\frac{\delta_1}{\delta_2} < \frac{\gamma_1}{\gamma_2}$.■

Proof of lemma 2:

This follows immediately from the condition for

$$E\left\{U\left(x_1 = \frac{\hat{v}_1 - p_{ask}}{\alpha\rho\hat{\Sigma}_1} + \frac{\beta}{\alpha}x_2\right)\right\} > E\{U(x_1 = 0)\} \qquad \text{(A 9)}$$

$$\Leftrightarrow (\alpha x_1 - \beta x_2)(\hat{v}_1 - p_{ask}) - \frac{\rho}{2}\{\alpha^2 x_1^2 - 2\alpha\beta x_1 x_2 + \beta^2 x_2^2\}\hat{\Sigma}_1 > -\beta x_2(\hat{v}_1 - p_{ask}) - \frac{\rho}{2}\beta^2 x_2^2 \hat{\Sigma}_1$$

$$\Leftrightarrow \alpha x_1(\hat{v}_1 - p_{ask}) - \frac{\rho}{2}\{\alpha^2 x_1^2 - 2\alpha\beta x_1 x_2\}\hat{\Sigma}_1 > 0$$

$$\Leftrightarrow (\hat{v}_1 - p_{ask}) - \frac{\rho}{2}\{\alpha x_1 - 2\beta x_2\}\hat{\Sigma}_1 > 0$$

$$\Leftrightarrow 2\{\frac{\hat{v}_1 - p_{ask}}{\alpha\rho\hat{\Sigma}_1} + \frac{\beta}{\alpha}x_2\} > x_1$$

$\Leftrightarrow 2x_1 > x_1^*$ which is always true. This completes the proof. ■

Proof of proposition 6:

Since the principal's objective is decreasing in α, the optimal α is the smallest positive root of:

$$\alpha E\pi_A - C - \frac{\rho}{2}\alpha^2 V_A = 0 \qquad \text{(A 10)}$$

Using the relationship $V_A = 2\,(E\pi_A)^2$ gives expression (6.9). The principal prefers herding if and only if:

$$E\pi_A - \frac{\rho}{2}\alpha^2 V_A < \frac{E\pi_A}{\sqrt{2}} \qquad \text{(A 11)}$$

which gives the condition in the text.■

Proof of proposition 7:

Inserting $\beta = \alpha C_{AB}/V_B$ gives:

$$\alpha^2 \ V_A - 2 \ \alpha \ \beta \ C_{AB} + \beta^2 \ V_B = \alpha^2 \ V_A \ (1 - R^2_{AB}) \qquad \text{(A 12)}$$

Using $V_A = 2\,(E(\pi_A))^2$ gives the desired result.■

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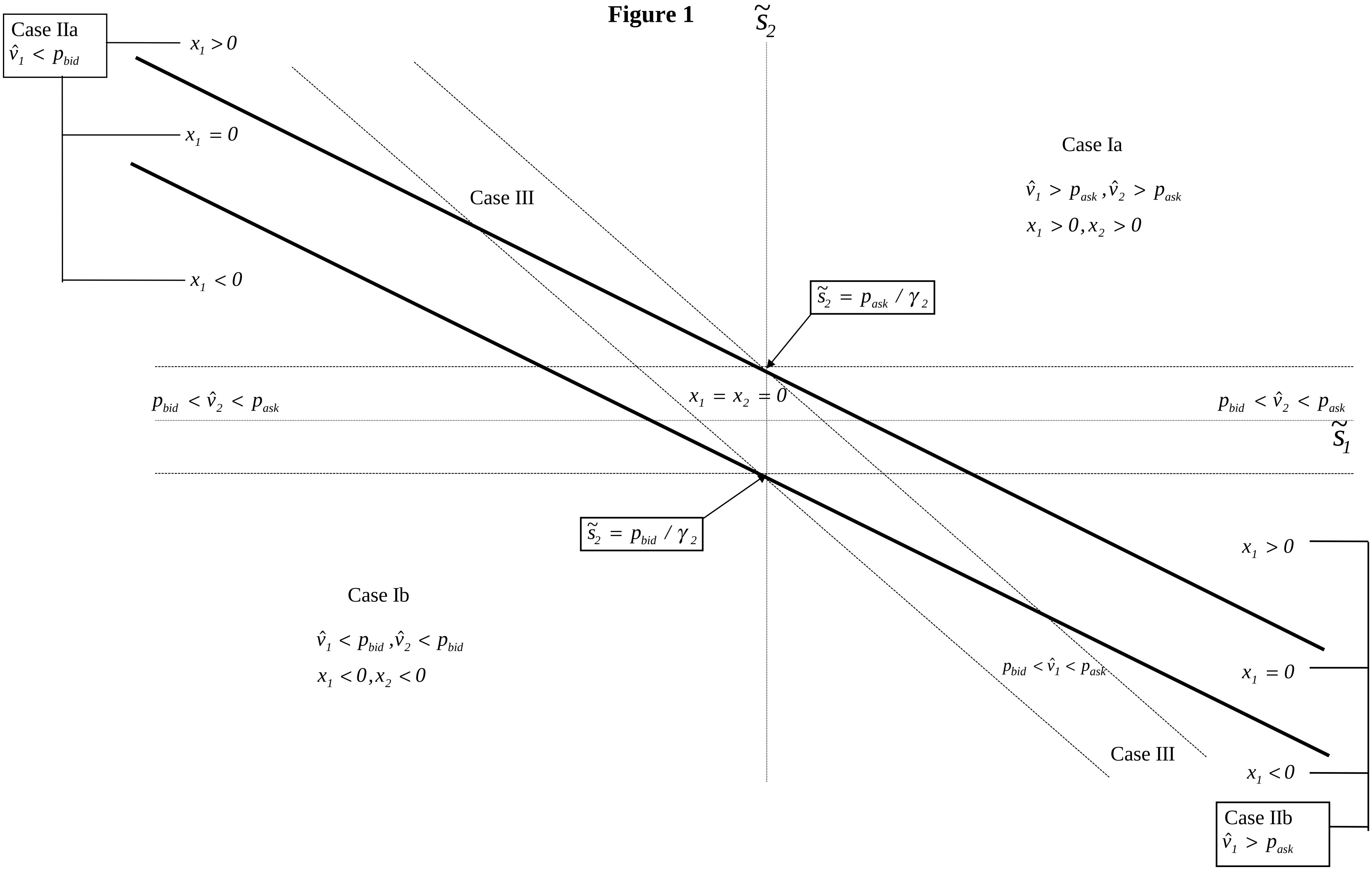
Figure 1



Case IIa
$\hat{v}_1 < p_{bid}$
$x_1 > 0$
$x_1 = 0$
$x_1 < 0$
$\tilde{s}_2$
Case III
Case Ia
$\hat{v}_1 > p_{ask}, \hat{v}_2 > p_{ask}$
$x_1 > 0, x_2 > 0$
$\tilde{s}_2 = p_{ask} / \gamma_2$
$p_{bid} < \hat{v}_2 < p_{ask}$
$x_1 = x_2 = 0$
$p_{bid} < \hat{v}_2 < p_{ask}$
$\tilde{s}_1$
$\tilde{s}_2 = p_{bid} / \gamma_2$
$x_1 > 0$
Case Ib
$\hat{v}_1 < p_{bid}, \hat{v}_2 < p_{bid}$
$x_1 < 0, x_2 < 0$
$p_{bid} < \hat{v}_1 < p_{ask}$
$x_1 = 0$
Case III
$x_1 < 0$
Case IIb
$\hat{v}_1 > p_{ask}$